OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. If $x^2 + kx + 6 = (x + 2)(x + 3)$ for all x, then the value of k is:

(a) 1 | **(b) –1**

(c) 5 | **(d) 3**

[NCERT Exemplar]

Ans. *(c) 5*

Explanation:

Given equation is $x^2 + kx + 6 = (x + 2)(x + 3)$

On simplifying,

$$x^2 + kx + 6 = (x + 2)(x + 3)$$
$$x^2 + kx + 6 = x^2 + 3x + 2x + 6$$
$$x^2 + kx + 6 = x^2 + 5x + 6$$

On comparing, we get

$$kx = 5x$$
$$k = 5$$

Therefore, the value of k is 5.

***2. The remainder if $x^4 + 8x + 5$ is divided by $x + 4$ is:**

(a) 229 | **(b) 134**

(c) –229 | **(d) –234** **[Diksha]**

Ans. *(a) 229*

Explanation: When we divide $x^4 + 8x + 5$ by $x + 4$, we get

$$
\begin{array}{r|l}
 & x^3 + 4x^2 + 16x - 56 \\
\hline
x + 4 & x^4 + 0x^3 + 0x^2 + 8x + 5 \\
 & x^4 + 4x^3 \\
 & - \quad - \\
\hline
 & -4x^3 + 0x^2 + 8x + 5 \\
 & -4x^3 - 16x^2 \\
 & + \quad + \\
\hline
 & 16x^2 + 8x + 5 \\
 & 16x^2 + 64x \\
 & - \quad - \\
\hline
 & -56x + 5 \\
 & -56x - 224 \\
 & + \quad + \\
\hline
 & 229
\end{array}
$$

***3. Using remainder theorem, find the remainder when $x^4 + 8x + 5$ is divided by $x + 4$.**

(a) 229 | **(b) 134**

(c) –229 | **(d) –234**

Ans. *(a) 229*

Explanation:

Let given polynomials be:

$$p(x) = x^4 + 8x + 5 \ldots\text{(i)}$$
$$g(x) = x + 4$$

For finding the zero of $g(x)$, put $g(x) = 0$

$$x + 4 = 0$$
$$\Rightarrow \quad x = -4$$

So, it is the zero of $g(x)$.

* *Topics and Questions which are a part of latest CBSE Syllabus but have been removed by NCERT.*

On putting $x = -4$ in equation (i), we get

$$\begin{aligned} p(-4) &= (-4)^4 + 8(-4) + 5 \\ &= 256 - 32 + 5 \\ &= 229 \end{aligned}$$

Hence, the value of $p(-4)$ is 229, which is the required remainder obtained by dividing $x^4 + 8x + 5$ by $x + 4$.

4. Which of the following option is a zero of the polynomial $3x^2 + 11x + 8$?

(a) $-\frac{8}{3}$ **(b)** $\frac{8}{3}$

(c) -3 **(d)** -2 **[Diksha]**

Ans. *(a)* $-\frac{8}{3}$

Explanation: Let $p(x) = 3x^2 + 11x + 8$

A real number 'a' is a zero of a polynomial $p(x)$ if $p(a) = 0$

Evaluate $P\left(-\frac{8}{3}\right)$

$$\begin{aligned} P\left(-\frac{8}{3}\right) &= 3\left(-\frac{8}{3}\right)^2 + 11\left(-\frac{8}{3}\right) + 8 \\ &= 3\left(\frac{64}{9}\right) - \frac{88}{3} + 8 \\ &= \frac{64}{3} - \frac{88}{3} + 8 \\ &= -\frac{24}{3} + 8 = -8 + 8 \\ &= 0 \end{aligned}$$

Therefore, $-\frac{8}{3}$ is a zero of polynomial

$3x^2 + 11x + 8$.

5. Which of the following options gives the correct value of the polynomial $x^3 - 10x^2 + 3x - 4$ at $x = -1$?

(a) -18 **(b)** 10

(c) 4 **(d)** 2 **[Diksha]**

Ans. *(a) −18*

Explanation: Let $p(x) = x^3 - 10x^2 + 3x - 4$

Now, at $x = -1$

$$\begin{aligned} p(-1) &= (-1)^3 - 10(-1)^2 + 3(-1) - 4 \\ &= -1 - 10 - 3 - 4 \\ &= -18 \end{aligned}$$

Hence, at $x = -1$, the polynomial $x^3 - 10x^2 + 3x - 4$ gives -18 as the correct value.

6. The value of $x^3 + y^3$, if $x + y = 15$ and $xy = 25$ is:

(a) 2250 **(b)** 1250

(c) 1750 **(d)** 3250

Ans. *(a) 2250*

Explanation:

We know,

$$x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$$

And $(x + y)^2 = x^2 + y^2 + 2xy$

Now, $x + y = 15$

Squaring both sides

$$\begin{aligned} x^2 + y^2 + 2xy &= 225 \\ x^2 + y^2 &= 225 - 2xy \\ &= 225 - 2(25) \\ &= 175 \end{aligned}$$

$$\begin{aligned} \therefore \quad x^3 + y^3 &= (x + y)(x^2 + y^2 - xy) \\ &= 15\,(175 - 25) \\ &= 15\,(150) \\ x^3 + y^3 &= 2250 \end{aligned}$$

***7. The remainder if $x^2 + px + q$ is divided by $x + r$ is:**

(a) $-r^2 - pr - q$ **(b)** $r^2 + pr + q$

(c) $r^2 - pr + q$ **(d)** $r^2 - pr - q$ **[Diksha]**

Ans. *(c)* $r^2 - pr + q$

Explanation: When we divide $x^2 + px + q$ by $x + r$ we get $r^2 - pr + q$

$$\begin{array}{r|l} & x + p - r \\ \hline x + r & x^2 + px + q \\ & x^2 + rx \\ & -\quad - \\ \hline & px - rx + q \\ & px + pr \\ & -\quad - \\ \hline & -rx - pr + q \\ & -rx - r^2 \\ & +\quad + \\ \hline & -pr + r^2 + q \end{array}$$

8. Which of these is a polynomial in one variable?

(a) The perimeter of a square whose length of side is represented by the expression $\sqrt{x}$.

(b) The area of a square whose length of side is represented by the expression $1 + \sqrt{x}$.

(c) The area of a rectangle whose length of side is represented by the expression $2 + \sqrt{x}$ and $\sqrt{x}$.

(d) The perimeter of a rectangle whose length of side is represented by the expression $x^2 + \sqrt{x}$ and $5 - \sqrt{x}$.

Ans. *(d) The perimeter of a rectangle whose length of side is represented by the expression $x^2 + \sqrt{x}$ and $5 - \sqrt{x}$.*

Explanation: Perimeter of a rectangle is $2(l_1 + l_2)$

Where, l_1 and l_2 are the length and breadth of the rectangle respectively.

$\therefore$ In option (d),

$$l_1 = x^2 + \sqrt{x} \text{ and } l_2 = 5 - \sqrt{x}$$

$$\text{Perimeter} = 2(l_1 + l_2)$$
$$= 2(x^2 + \sqrt{x} + 5 - \sqrt{x})$$
$$= 2(x^2 + 5);$$

which is a polynomial in one variable.

9. If $x + y = 14$ and $xy = 24$, then the value of $x^2 - xy + y^2$ is:

(a) 196 **(b) 124**

(c) –142 **(d) 72**

Ans. *(b) 124*

Explanation:

We know,

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2) \quad ...(i)$$

And $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

We can write,

$$x^3 + y^3 = (x + y)^3 - 3xy(x + y) \quad ...(ii)$$

Now, equating equations (i) and (ii)

$$(x + y)(x^2 - xy + y^2) = (x + y)^3 - 3xy(x + y)$$

$$(x^2 - xy + y^2) = \frac{(x+y)^3 - 3xy(x+y)}{(x+y)}$$
$$= \frac{(x+y)[(x+y)^2 - 3xy]}{(x+y)}$$
$$= (x + y)^2 - 3xy$$
$$= (14)^2 - 3(24)$$
$$= 196 - 72$$
$$= 124$$

10. If $x + \frac{1}{x} = \sqrt{3}$, then the value of $x^3 + \frac{1}{x^3}$ is:

(a) 0 **(b) 1**

(c) 2 **(d) 3**

Ans. *(a) 0*

Explanation:

We know,

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

Now, $\left(x + \frac{1}{x}\right) = \sqrt{3}$

Cubing both sides, we get

$$x^3 + \frac{1}{x^3} + 3\left(x \times \frac{1}{x}\right)\left(x + \frac{1}{x}\right) = 3\sqrt{3} \quad ...(i)$$

Now put, $x + \frac{1}{x} = \sqrt{3}$ in equation (i)

$$x^3 + \frac{1}{x^3} + 3\sqrt{3} = 3\sqrt{3}$$

$$x^3 + \frac{1}{x^3} = 3\sqrt{3} - 3\sqrt{3}$$

$$x^3 + \frac{1}{x^3} = 0$$

11. $x + 1$ is a factor of the polynomial:

(a) $x^3 + x^2 - x + 1$ **(b) $x^3 + x^2 + x + 1$**

(c) $x^4 + x^3 + x^2 + 1$ **(d) $x^4 + 3x^3 + 3x^2 + x + 1$**

[NCERT Exemplar]

Ans. *(b) $x^3 + x^2 + x + 1$*

Explanation: For $x + 1$ to be a factor as a polynomial, it should have remainder as zero on dividing the polynomial.

$$p(x) = x^3 + x^2 + x + 1$$

On dividing $p(x)$ by $x + 1$

$$x + 1 \overline{)x^3 + x^2 + x + 1}(x^2 + 1$$
$$\underline{x^3 + x^2}$$
$$- \quad -$$
$$0 + x + 1$$
$$\underline{x + 1}$$
$$- \quad -$$
$$0$$

Hence, $x + 1$ is a factor of $x^3 + x^2 + x + 1$.

12. One of the factors of $(25x^2 - 1) + (1 + 5x)^2$ is:

(a) $5 + x$ **(b) $5 - x$**

(c) $5x - 1$ **(d) $10x$**

Ans. *(d) $10x$*

Explanation:

Let $p(x) = (25x^2 - 1) + (1 + 5x)^2$

On simplifying,

$$p(x) = 25x^2 - 1 + (1 + 5x)^2$$
$$p(x) = 25x^2 - 1 + (1 + 10x + 25x^2)$$
$$p(x) = 25x^2 - 1 + 1 + 10x + 25x^2$$
$$p(x) = 50x^2 + 10x$$
$$p(x) = 10x(5x + 1)$$

Here, $10x$ and $(5x + 1)$ are two factors of $p(x)$.

Hence, $10x$ is one of the factor of $(25x^2 - 1) + (1 + 5x)^2$.

13. The value of $9 - 36(a + b)^2$ is:

(a) $9(1 + 2a - 2b)(1 + 2a + 2b)$

(b) $9(1 - a - 2b)(1 - a + 2b)$

(c) $9(1 + 2a + 2b)(1 - 2a + 2b)$

(d) $9(1 + 2a + 2b)(1 - 2a - 2b)$

Ans. *(d) $9(1 + 2a + 2b)(1 - 2a - 2b)$*

Explanation: Given equation is

$$= 9 - 36(a + b)^2$$
$$= 9[1 - 4(a + b)^2]$$
$$= 9[1^2 - (2(a + b))^2]$$
$$= 9[(1 + 2(a + b)][1 - 2(a + b)]$$
$$= 9(1 + 2a + 2b)(1 - 2a - 2b)$$

14. One of the zeroes of the polynomial $12x^3 - 24x^2 - 4x + 8$ is ....

(a) $-\frac{2}{\sqrt{12}}$ (b) -2

(c) 1 (d) 12 [Diksha]

Ans. *(a)* $-\frac{2}{\sqrt{12}}$

Explanation: Let $p(x) = 12x^3 - 24x^2 - 4x + 8$

Evaluate $p(x)$ for the values given in the options

$$p\left(-\frac{2}{\sqrt{12}}\right) = 12\left(-\frac{2}{\sqrt{12}}\right)^3 - 24\left(-\frac{2}{\sqrt{12}}\right)^2 - 4\left(-\frac{2}{\sqrt{12}}\right) + 8 = 0$$

$p(-2) = 12(-2)^3 - 24(-2)^3 - 4(-2) + 8 = -176$

$p(1) = 12(1)^3 - 24(-2)^2 - 4(1) + 8 = -8$

$p(12) = 12(12)^3 - 24(12)^2 - 4(12) + 8 = 17.240$

Only in option (a) *i.e.*, $p\left(-\frac{2}{\sqrt{12}}\right)$ we are getting the volue as zero hence $-\frac{2}{\sqrt{12}}$ is one of the zeroes of $12x^3 - 24x^2 - 4x + 8$.

15. The polynomial $(x - a)$ is a factor of the polynomial $x^4 - 2x^2 + kx + k$, where k is a constant. Which of these is the correct relation between a and k?

(a) $k = \frac{a^2(2-a^2)}{a+1}$ (b) $k = \frac{a^2(2+a^2)}{1+a}$

(c) $k = \frac{a^2(2-a^2)}{1-a}$ (d) $k = \frac{a^2(2-a^2)}{1-a}$

Ans. *(a)* $k = \frac{a^2(2-a^2)}{a+1}$

Explanation:

Let, $p(x) = x^4 - 2x^2 + kx + k$

If $x - a$ is a factor of $p(x)$, then $p(a) = 0$

$$(a)^4 - 2(a)^2 + ak + k = 0$$

$$a^4 - 2a^2 = -k(a + 1)$$

$$-k = \frac{a^4 - 2a^2}{a+1}$$

$$k = \frac{2a^2 - a^4}{a+1}$$

$$k = \frac{a^2\ (2-a^2)}{(a+1)}$$

***16.** The remainder when $8x^2 - 5x + 1$ is divided by $x - 2$ is:

(a) -1 (b) 0

(c) 21 (d) 23 [Diksha]

Ans. *(d)* 23

Explanation: Let $p(x) = 8x^2 - 5x + 1$

The zero of the polynomial $x - 2$ is 2

$$\begin{array}{r|l} & 8x + 11 \\ x-2 & 8x^2 - 5x + 1 \\ & \underline{8x^2 - 16x} \\ & \quad\ \ {}_{-\quad +} \\ & \quad 11x + 1 \\ & \quad \underline{11x - 22} \\ & \quad\ \ {}_{-\quad +} \\ & \qquad 23 \end{array}$$

So, by the Remainder Theorem, 23 is the remainder when $8x^2 - 5x \div 1$ is divided by $x - 2$.

Alternatively, we can do the long division. When we divide $8x^2 - 5x + 1$ by $x - 2$, we get 23 as the remainder.

Assertion and Reason (A–R)

Direction for questions 17 to 20: In question number 17 to 20, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct option as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

17. Assertion (A): The degree of the polynomial $6a^2 - \frac{1}{8}a^6 + 34a^3 - \sqrt{12}\,a$ is 6.

Reason (R): The degree of a polynomial is the highest power of its variable.

Ans. *(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).*

Explanation:

Given: $6a^2 - \frac{1}{8}a^6 + 34a^3 - \sqrt{12}\,a$

Here, the degree of this polynomial is 6 because the degree of a polynomial is the highest power of its variable.

18. Assertion (A): The factorisation of $z^3 + 125$ is $(z + 5)(z^2 - 5z + 25)$.

Reason (R): We know $x^3 + y^3 = (x + y)^3 - 3xy(x + y)$

Ans. *(c) Assertion (A) is true but reason (R) is false.*

Explanation:

We know, $x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$

$z^3 + (5)^3 = (z + 5)(z^2 + 25 - 5z)$

* *Topics and Questions which are a part of latest CBSE Syllabus but have been removed by NCERT.*

19. Assertion (A): The value of $x^2 + y^2 + 4z^2 + 2xy + 4yz + 4zx$, when $(x + y) = 5$ and $z = 3$ is 121.

Reason (R): If $x + \frac{1}{x} = 3$, then $x^2 + \frac{1}{x^2} = 7$.

Ans. (b) *Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).*

Explanation: We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$.....(i)

Given, $x^2 + y^2 + 4z^2 + 2xy + 4yz + 4zx$

$= x^2 + y^2 + (2z)^2 + 2xy + 2(y)(2z) + 2(2z)(x)$

When compare with equation (i)

$a = x$

$b = y$

$c = 2z$

We get,

$(x + y + 2z)^2 = x^2 + y^2 + 4z^2 + 2xy + 4yz + 4zx$

$[(5 + 2(3))]^2 = x^2 + y^2 + 4z^2 + 2xy + 4yz + 4zx$

$x^2 + y^2 + 4z^2 + 2xy + 4xy + 4yz + 4zx = (5 + 6)^2 = 121$

So, Assertion (A) is correct.

Now, $x + \frac{1}{x} = 3$

Squaring both sides, we get

$$x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} = 9$$

$$x^2 + \frac{1}{x^2} + 2 = 9$$

$$x^2 + \frac{1}{x^2} = 7$$

20. Assertion (A): $f(x) = 2 - x^2 + x^3$ is a cubic polynomial.

Reason (R): Every polynomial is a binomial.

Ans. (c) *Assertion (A) is true but reason (R) is false.*

Explanation: $f(x) = 2 - x^2 + x^3$

A polynomial of degree 3 is a cubic polynomial, and here $f(x)$ has degree 3.

A polynomial can be of any degree.

CASE BASED Questions (CBQs)

[**4** & **5** marks]

Read the following passages and answer the questions that follow:

21. *An object which is thrown or projected into the air, subject to only the acceleration of gravity is called a projectile and its path is called its trajectory. The curved path a projectile follows was shown by Galileo to be a parabola. Parabola is represented by a polynomial. If the polynomial to represent the distance covered is:* $p(x) = -6x^2 + 48x + 24$.

(A) What is the degree of the polynomial?

(a) 0 (b) 1

(c) 2 (d) 3

(B) Find the height of the projectile 5 seconds after it's launched.

(a) 114 m (b) 64 m

(c) 170 m (d) 116 m

(C) The value of K if $x - 3$ is a factor of $K^2x^3 - x^2 + 3x - 1$, is:

(a) 0 (b) $\pm\frac{1}{3\sqrt{3}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

(D) If the polynomial to represent the distance covered is given by, $p(x) = -6x^2 + 48x + 24$, then the value of $p(4)$ is:

(a) 117 (b) −72

(c) 120 (d) 114

(E) If the equation of the parabola is given by, $p(y) = 3y^2 - 2y - 4$, then the value of $p(2)$ is:

(a) 8 (b) 4

(c) 12 (d) 6

Ans. (A) *(c) 2*

Explanation: The degree of the polynomial is 2.

(B) *(a) 114 m*

Explanation: $p(y) = -6y^2 + 48y + 24$

To calculate height, put $y = 5$

Height of projectile,

$$p(5) = -6 \times (5)^2 + 48 \times 5 + 24$$
$$= -6 \times 25 + 240 + 24$$
$$= 114$$

So, the height of projectile reaches is 114 m.

(C) *(b)* $\pm\frac{1}{3\sqrt{3}}$

Explanation: Let $p(x) = K^2x^3 - x^2 + 3x - 1$.

If $(x - 3)$ is a factor of $p(x)$ then $p(x) = 0$.

$$\therefore \quad p(3) = K^2(3)^3 - (3)^2 + 3(3) - 1$$
$$= K^2(27) - 9 + 9 - 1$$
$$= 27K^2 - 1$$
$$K^2 = \frac{1}{27}$$
$$K^2 = \pm\frac{1}{3\sqrt{3}}$$

(D) *(c) 120*

Explanation: $p(x) = 6x^2 + 48x + 24$

Put $x = 4$

$$\therefore \quad p(4) = 6(4)^2 + 48(4) + 24$$
$$= -96 + 192 + 24$$
$$= 120$$

Hence, the value of $p(4)$ is 120.

(E) *(b) 4*

Explanation: $p(y) = 3y^2 - 2y - 4$

$$p(2) = 3 \times (2)^2 - 2 \times 2 - 4$$
$$= 3 \times 4 - 4 - 4$$
$$= 12 - 8$$
$$p(2) = 4$$

22. *Sonu's elder sister gave him some money to buy oranges from the fruit market at the rate of $p(y) = y^2 - 5y + 6$, where, α, β are the zeroes of $p(y)$.*

(A) Find α and β where $\alpha > \beta$.

(B) Find the value of $\alpha + \beta + \alpha\beta$ and $p(5)$

(C) Find the value of $\alpha^2 - \beta^2$

Ans. (A) Given polynomial $p(y) = y^2 - 5y + 6$

$$p(y) = y^2 - 3y - 2y + 6$$
$$p(y) = y(y - 3) - 2(y - 3)$$
$$p(y) = (y - 3)(y - 2)$$
$$y - 3 = 0 \qquad y - 2 = 0$$
$$y = 3 \qquad y = 2$$

As according to the question $\alpha > \beta$

So, $\alpha = 3$

As according to the question $\alpha > \beta$

So, $\beta = 2$

(B) Here, $\alpha = 3, \beta = 2$

$$\alpha + \beta + \alpha\beta = 3 + 2 + 3 \times 2$$
$$= 11$$
$$p(y) = y^2 - 5y + 6$$
$$p(5) = (5)^2 - 5 \times 5 + 6$$
$$= 25 - 25 + 6$$
$$p(5) = 6$$

(C) Here, $\alpha = 3$, $\beta = 2$

$$\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta)$$
$$= (3 + 2)(3 - 2)$$
$$= 5$$

23. *Junk food is a food that contains high levels of salt, sugar, fats and lack of nutrients such as vitamins, fibre and minerals, consuming them can lead to short and long-term health complications, including weight gain. If α be the number of children who take junk food and β be the number of children who take healthy food such that $\alpha > \beta$ where α and β are the zeroes of the quadratic polynomial*

$p(y) = 2y^2 - 18y + 40$

(A) Find the number of students who take healthy food.

(B) How many students take junk food?

(C) Find the value of k, if $p(0) + p(1) = k.\ p(2)$.

Ans. (A)

$$p(y) = 2y^2 - 18y + 40$$
$$= 2y^2 - 10y - 8y + 40$$
$$= 2y(y - 5) - 8(y - 5)$$
$$= (2y - 8)(y - 5)$$
$$2y - 8 = 0, \quad y - 5 = 0$$
$$2y = 8, \quad y = 5$$
$$y = 4$$

As $\alpha > \beta$, so $\beta = 4$

$\therefore$ Number of students who take healthy food = 4

(B)

$$p(y) = 2y^2 - 18y + 40$$
$$= 2y^2 - 10y - 8y + 40$$

$= 2y(y - 5) - 8(y - 5)$

$= (2y - 8)(y - 5)$

$2y - 8 = 0, \quad y - 5 = 0$

$2y = 8, \quad y = 5$

$y = 5$

As $\alpha > \beta$, So $\alpha = 5$

∴ Number of students who take junk food = 5

(C) $p(y) = 2y^2 - 18y + 40$

$p(0) = 2(0) - 18 \times 0 + 40$

$p(0) = 40$

$p(1) = 2(1)^2 - 18(1) + 40$

$= 2 - 18 + 40$

$= 42 - 18$

$= 24$

$p(2) = 2(2)^2 - 18(2) + 40$

$= 2 \times 4 - 36 + 40$

$= 48 - 36$

$= 12$

$p(0) + p(1) = k.\ p(2)$

$40 + 24 = k.(12)$

$64 = k.(12)$

$k = \frac{64}{12}$

$k = \frac{16}{3}$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

24. If $p(x) = 2x^3 - 6x^2 + ax + a$, and $(x + 2)$ is a factor of $p(x)$, then find a.

Ans. $p(x) = 2x^3 - 6x^2 + ax + a$

If $(x + 2)$ is a factor, $p(-2) = 0$

$p(-2) = 0$

$2(-2)^3 - 6(-2)^2 + a(-2) + a = 0$

$\Rightarrow \quad 2(-8) - 6(4) - 2a + a = 0$

$\Rightarrow \quad -16 - 24 - a = 0$

$a = -40$

25. Find the zero of polynomial in each of the follwing cases:

(A) $p(x) = 2x + 5$

(B) $g(x) = 3 - 6x$ [NCERT Exemplar]

Ans. (A) Given, $p(x) = 2x + 5$

$p(x) = 0$

$2x + 5 = 0$

$2x = -5$

$x = \frac{-5}{2}$

(B) Given, $g(x) = 3 - 6x$

$g(x) = 0$

$3 - 6x = 0$

$x = \frac{3}{6}$

$x = \frac{1}{2}$

***26. If $x^{111} + 111$ is divided by $x + 1$, then find the remainder.**

Ans. Given, divisor is $x + 1$, then by remainder theorem, $x + 1 = 0$

$x = -1$

If $p(x) = x^{111} + 111$

$p(-1) = (-1)^{111} + 111$

$= -1 + 111$

$= 110$

Hence, the remainder is 110.

27. If $a + b + c = 0$, then find $a^3 + b^3 + c^3$. [Diksha]

Ans. We know,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Given, $a + b + c = 0$

$\therefore a^3 + b^3 + c^3 - 3abc = 0(a^2 + b^2 + c^2 - ab - bc - ca)$

$a^3 + b^3 + c^3 - 3abc = 0$

$a^3 + b^3 + c^3 = 3abc$

Hence, the value of $a^3 + b^3 + c^3 = 3abc$

28. The volume of a cube is given by the expression $27x^3 + 8y^3 + 54x^2y + 36xy^2$. What is the expression for the length of a side of the cube?

Ans. The volume of the cube is given by

$p(x) = 27x^3 + 8y^3 + 54x^2 y + 36xy^2$

We know,

volume of cube = $(\text{side})^3$

* *Topics and Questions which are a part of latest CBSE Syllabus but have been removed by NCERT.*

So, $p(x) = 27^{x3} + 8y^3 + 54x^2y + 36xy^2$

$p(x) = (3x)^3 + (2y)^3 + 3 \times (3x)(2y)(3x + 2y)$

$p(x) = (3x + 2y)^3$

So, $p(x) = (3x + 2y)^3 = (\text{Side})^3$

Thus, the side of the cube is $(3x + 2y)$ units.

Concept applied

➥ $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

29. Find the value of $x - \frac{1}{x}$, if $x + \frac{1}{x} = 3$.

Ans. Given: $x + \frac{1}{x} = 3$

Squaring both sides, we get

$$x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} = 9$$

$$x^2 + \frac{1}{x^2} = 7$$

Subtract $-2 \times x \times \frac{1}{x}$ both the sides.

$$x^2 + \frac{1}{x^2} - 2x \times \frac{1}{x} = 7 - 2 \times x \times \frac{1}{x}$$

$$(x - \frac{1}{x})^2 = 5$$

$$x - \frac{1}{x} = \pm\sqrt{5}$$

30. Factorise $84 - 2r - 2r^2$ [NCERT Exemplar]

Ans. Given: $84 - 2r - 2r^2$

$-2[r^2 + r - 42] = 0$

$\Rightarrow -2[r^2 + 7r - 6r - 42] = 0$

[Factorising by splitting the middle term]

$\Rightarrow -2[r(r + 7) - 6(r + 7)] = 0$

$\Rightarrow -2[(r - 6)(r + 7)] = 0$

$\Rightarrow -2[-(6 - r)(r + 7)] = 0$

$\Rightarrow 2[(6 - r)(7 + r)] = 0$

SHORT ANSWER Type-I Questions (SA-I)

[**2** marks]

31. Check whether $(r+1)$ is a factor of polynomial $(4r^3 + 4r^2 - r - 1)$.

Ans. Let, $p(r) = 4r^3 + 4r^2 - r - 1$

And $q(r) = r + 1$

On putting $q(r) = 0$, we get

$r + 1 = 0$

$r = -1$

On puttingr $= -1$ in $p(r)$, we get

$p(-1) = 4(-1)^3 + 4(-1)^2 - (-1) - 1$

$= -4 + 4 + 1 - 1$

$= 0$

Since, $p(-1) = 0$

Therefore, $(r + 1)$ is a factor of $(4r^3 + 4r^2 - r - 1)$

32. Ashima donated a certain amount of money to a blind school. Her friend Manya wanted to know the amount donated by her, but Ashima did not disclose the amount she donated, instead she gave her a hint that if $\left(x + \frac{1}{x}\right)$ = ₹7 then the amount donated by her is ₹ $\left(x^3 + \frac{1}{x^3}\right)$. Find the amount donated by Ashima to the school.

[British Council 2022]

Ans. $\because \quad x + \frac{1}{x} = 7$

$\therefore \quad \left[x + \frac{1}{x}\right]^3 = 7^3$

$\Rightarrow \quad x^3 + \frac{1}{x^3} + 3x\frac{1}{x}\left(x + \frac{1}{x}\right) = 343$

$\Rightarrow \quad x^3 + \frac{1}{x^3} + 3 \times 1 \times (7) = 343$

$\Rightarrow \quad x^3 + \frac{1}{x^3} + 21 = 343$

$\Rightarrow \quad x^3 + \frac{1}{x^3} = 343 - 21$

$= 322$

Thus, the amount donated by Ashima is ₹ 322.

33. The value of the polynomial $x^2 + kx + 5$ at $x = 2$ is 15 where k is a constant. Find the value of the polynomial at $x = 5$. [NCERT Exemplar]

Ans. Let, $p(x) = x^2 + kx + 5$

Now, at $x = 2$

$p(2) = 15$ [Given]

$(2)^2 + k(2) + 5 = 15$

$4 + 2k + 5 = 15$

$$2k = 15 - 9$$

$$k = \frac{6}{2}$$

$$k = 3$$

Hence, $p(x) = x^2 + 3x + 5$

Now, put $x = 5$, we get

$$p(5) = (5)^2 + 3(5) + 5$$

$$= 25 + 15 + 5$$

$$p(5) = 45$$

34. Find the value of k if $x - 2$ is a factor of $4x^3 + 3x^2 - 4x + k$. **[Delhi Gov. QB 2022]**

Ans. $x - 2$ is factor of $4x^3 + 3x^2 - 4x + k$

$$x - 2 = 0$$

$$x = 2$$

$$4(2)^3 + 3(2)^2 - 4x \times 2 + k = 0$$

$$32 + 12 - 8 + k = 0$$

$$44 - 8 + k = 0$$

$$36 + k = 0$$

$$k = -36$$

35. If $a + b + c = 9$ and $ab + bc + ca = 26$, find $a^2 + b^2 + c^2$.

Ans. Given, $a + b + c = 9$ and $ab + bc + ca = 26$

$\Rightarrow$ $(a + b + c)^2 = (9)^2$

[$\because$ Squaring on the both sides]

$\Rightarrow$ $a^2 + b^2 + c^2 + 2(ab + bc + ca) = 81$

$\Rightarrow$ $a^2 + b^2 + c^2 + 2(26) = 81$

[$\because ab + bc + ca = 26$]

$\Rightarrow$ $a^2 + b^2 + c^2 = 81 - 52$

$= 29$

36. Factorise $64(x + y)^3 - 125(x - y)^3$.

Ans. $64(x + y)^3 - 125(x - y)^3 = [\{4(x + y)\}^3 - \{5(x - y)\}^3]$

we know, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$= \{4(x + y) - 5(x - y)\}[16(x + y)^2 + 20(x + y)(x - y) + 25(x - y)^2]$

$= (9y - x)[16(x + y)^2 + 20(x + y)(x - y) + 25(x - y)^2]$

$= (9 - x)[16x^2 + 16y^2 + 32xy + 20x^2 - 20y^2 + 25x^2 + 25y^2 - 50xy]$

$= (9y - x)(61x^2 + 21y^2 - 18xy)$

SHORT ANSWER Type-II Questions (SA-II)

[**3** marks]

37. If polynomials $4x^3 + 2ax + 6x - 10$ and $3x^3 + 3x^2 - 12x + 3a$ leave the same remainder when divided by $(2x - 4)$, find the value of a.

Ans. Let, $p(x) = 4x^3 + 2ax + 6x - 10$ and $q(x) = 3x^3 + 3x^2 - 12x + 3a$ be the given polynomials. The remainder when divided by $2x - 4$ are $p(2)$ and $q(2)$, respectively.

By the given condition, we have

$$p(2) = q(2)$$

$$4(2)^3 + 2(a)(2) + 6 \times 2 - 10 = 3(2)^3 + 3(2)^2 - 12 \times 2 + 3a$$

$$32 + 4a + 12 - 10 = 24 + 12 - 24 + 3a$$

$$34 + 4a = 12 + 3a$$

$$34 - 12 = 3a - 4a$$

$$a = -22$$

38. (A) Find the value of $48^3 - 30^3 - 18^3$ using suitable algebraic identities.

(B) Factorise $(x - y)^3 + (y - z)^3 + (z - x)^3$ using suitable algebraic identities.

Ans. (A) To find the value of $48^3 - 30^3 - 18^3$.

Since, $x + y + z = 0$,

then, $x^3 + y^3 + z^3 - 3xyz = 0$ or $x^3 + y^3 + z^3 = 3xyz$

Here, in $48^3 + (-30)^3 + (-18)^3$,

the value of sum, $48 + (-30) + (-18) = 0$

Therefore, $48^3 + (-30)^3 + (-18)^3$

$= 3 \times 48 \times (-30) \times (-18) = 77760$

(B) Given $(x - y)^3 + (y - z)^3 + (z - x)^3$

Since, $x + y + z = 0$,

then, $x^3 + y^3 + z^3 - 3xyz = 0$ or $x^3 + y^3 + z^3 = 3xyz$

Here, $(x - y) + (y - z) + (z - x) = 0$

Therefore, $(x - y)^3 + (y - z)^3 + (z - x)^3$

$= 3(x - y)(y - z)(z - x)$

39. For the polynomial

$\frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6$, write:

(A) the degree of the polynomial

(B) the coefficient of x^3

(C) the coefficient of x^6

Ans. Here, $\frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6$

$\Rightarrow$ $-x^6 + \frac{1}{5}(x^3 + 2x + 1) - \frac{7}{2}x^2$

$\Rightarrow$ $-x^6 + \frac{1}{5}x^3 - \frac{7}{2}x^2 + \frac{2}{5}x + \frac{1}{5}$

(A) The degree of the polynomial $\frac{x^3+2x+1}{5}-\frac{7}{2}x^2 - x^6$ is 6 because the maximum power of the x is 6.

(B) The coefficient of x^3 in $-x^6+\frac{1}{5}x^3-\frac{7}{2}x^2+\frac{2}{5}x+\frac{1}{5}$ is $\frac{1}{5}$

(C) The coefficient of x^6 in $-x^6+\frac{1}{5}x^3-\frac{7}{2}x^2+\frac{2}{5}x+\frac{1}{5}$ is -1

40. Factorise $x^{24} - y^{24}$.

Ans. $x^{24}-y^{24} = (x^{12})^2-(y^{12})^2$

$\Rightarrow \quad x^{24}-y^{24}$

$\Rightarrow \quad (x^{12}+y^{12})(x^{12}-y^{12})$

$= [(x^4)^3+(y^4)^3][(x^6)^2-(y^6)^2]$

$= x^4+y^4)(x^8+y^8-x^4y^4)[(x^6+y^6)(x^6-y^6)]$

$= (x^4+y^4)(x^8+y^8-x^4y^4)(x^2+y^2)[(x^2)^2-x^2y^2+(y^2)^2][(x^3+y^3)(x^3-y^3)]$

$= (x^4+y^4)(x^8+y^8-x^4y^4)(x^2+y^2)[(x^2)^2-x^2y^2+(y^2)^2][(x+y)[(x^2-xy+y^2](x-y)(x^2+xy+y^2)$

41. Find the length and breadth of a rectangle, if its area is represented by the polynomial, $6x^2 - 29x + 30$ and also find the perimeter.

Ans. We know, the area of rectangle $= l \times b$

As $6x^2-29x+30 = \text{Area} = l \times b$

Use middle term splitting method

$6x^2-29x+30 = l \times b$

$6x^2-20x-9x+30 = l \times b$

$2x(3x-10)-3(3x-10) = l \times b$

$(2x-3)(3x-10) = l \times b$

So, we have

$l = 2x-3$ and $b = 3x-10$

Now, perimeter $= 2(l+b]$

$= 2[2x-3+3x-10]$

$= 2[5x-13]$

Perimeter $= 10x-26$

42. Find the value of b.

(A) $81a^2 - b^2 = (9a + \frac{1}{2})(9a - \frac{1}{2})$

(B) $x^3 - 2bx^2 + 16$ is divisible by $x + 2$.

[Delhi Gov. QB 2022]

Ans. (A) Here, $81a^2-b^2 = (9a+\frac{1}{2})(9a-\frac{1}{2})$

$= (9a)^2-(\frac{1}{2})^2$

So, $81a^2-b^2 = 81a^2-\frac{1}{4}$

On comparing both sides, we get

$-b^2 = -\frac{1}{4}$

$b^2 = \frac{1}{4}$

$b = \pm\frac{1}{2}.$

Therefore, the value of b is $\pm\frac{1}{2}$.

(B) Consider, $p(x) = x^3-2bx^2+16$

When $p(x)$ is divisible by $(x+2)$, then remainder = 0

So $p(-2) = 0$

$(-2)^3-2b(-2)^2+16 = 0$

$-8-8b+16 = 0$

$\Rightarrow \quad -8b+8 = 0$

$\Rightarrow \quad -8b = -8$

$\Rightarrow \quad b = 1$

Therefore, the value of b is 1.

43. If $x - 3$ and $x - \frac{1}{3}$ are both factors of $px^2 + 5x + r$, then show that $p = r$.

Ans. $x-3$ and $x-\frac{1}{3}$ are factors of px^2+5x+r

$\therefore \quad x = 3, x = \frac{1}{3}$

Put $x = 3$

$\therefore \quad p(3)^2+5\times 3+r = 0$

$9p+15+r = 0$

$9p+r = -15 \quad$ ...(i)

Put $x = \frac{1}{3}$

$p\left(\frac{1}{3}\right)^2+5\times\left(\frac{1}{3}\right)+r = 0$

$\frac{p}{9}+\frac{5}{3}+r = 0$

$\frac{p+15+9r}{9} = 0$

$p+9r = -15 \quad$ ...(ii)

From eqs. (i) and (ii)

$9p+r = p+9r$

$9p-p = 9r-r$

$8p = 8r$

$p = r$

44. Find the values of m and n if the polynomial $2x^3 + mx^2 + nx - 14$ has $x - 1$ and $x - 2$ as its factors. [Delhi Gov. QB 2022]

Ans. $x - 1$ and $x + 2$ are factor of $2x^3 + mx^2 + nx - 14$

$x = 1, x = 2$

For $x = 1$,

$$2(1)^3 + m(1)^2 + n(1) - 14 = 0$$
$$2 + m + n - 14 = 0$$
$$m + n - 12 = 0$$
$$m + n = 12 \quad ...(i)$$
$$2(2)^3 + m(2)^2 + n(2) - 14 = 0$$
$$16 + 4m + 2n - 14 = 0$$
$$4m + 2n + 2 = 0$$
$$4m + 2n = -2$$
$$2m + n = -1 \quad ...(ii)$$

Subtracting eq. (ii) from eq. (i)

$$m = -13$$

Put $m = -13$ in eqs. (i), we get

$$-13 + n = 12$$
$$n = 12 + 13 = 25$$

LONG ANSWER Type Questions (LA)

[**4** & **5** marks]

45. $2x^3 + 4x^2 - 7ax - 5$ and $2x^3 + ax^2 - 6x + 3$ are polynomials which on dividing by $(x + 1)$ and $(x - 1)$ leaves remainders y and z, respectively, if $y - 3z = 16$ then find a.

Ans. Let, $p(x) = 2x^3 + 4x^2 - 7ax - 5$ and $q(x) = 2x^3 + ax^2 - 6x + 3$ be the given polynomials.

Now,

When $p(x)$ is divided by $(x + 1)$, remainder $= y$

$$y = p(-1)$$
$$y = 2(-1)^3 + 4(-1)^2 - 7a(-1) - 5$$
$$y = -2 + 4 + 7a - 5$$
$$y = -3 + 7a$$

And, when $q(x)$ is divided by $(x - 1)$, remainder $= z$

$$z = q(1)$$
$$z = 2(1)^3 + a(1)^2 - 6(1) + 3$$
$$z = 2 + a - 6 + 3$$
$$z = a - 1$$

Substituting the values of y and z, we have

$$y - 3z = 16$$
$$-3 + 7a - 3(a - 1) = 16$$
$$-3 + 7a - 3a + 3 = 16$$
$$4a = 16$$
$$a = 4$$

46. Two brothers Ashish and Amit wanted to start a business together. They decided to share their amount depending upon the variable expenditure. The amount of two partners is given by the expression $12x^2 + 11x - 15$, which is the product of their individual share factors.

(A) Find total expenditure of Ashish and Amit when $x =$ ₹ 100.

(B) Find individual share factor of Ashish and Amit in terms of x.

(C) Find the value of x if their shares are equal. [British Council 2022]

Ans. (A) Total expenditure $= 12x^2 + 11x - 15$

Put $x = 100$

$$= 12 \times 100 \times 100 + 1100 - 15$$
$$= 120000 + 1100 - 15$$
$$= ₹121{,}085$$

(B) $12x^2 + 11x - 15$

$$= 12x^2 + 20x - 9x - 15$$
$$= 4x(3x + 5) - 3(3x + 5)$$
$$= (3x + 5)(4x - 3)$$

Share of Ashish and Amit are either $(3x + 5)$ and $(4x - 3)$ or $(4x - 3)$ and $(3x + 5)$ respectively.

(C) According to the question, if their shares are equal

$$4x - 3 = 3x + 5$$
$$4x - 3x = 5 + 3$$
$$x = 8$$

47. If $4x - 2y = 4$ and $xy = \frac{6}{4}$, then find the value of $x^3 - \frac{y^3}{8}$.

Ans. Given: $4x - 2y = 4$

$$2x - y = 2$$

$$\Rightarrow \quad x - \frac{y}{2} = 1 \quad ...(i)$$

And $$xy = \frac{6}{4}$$
$$xy = \frac{3}{2}$$

Now we know,

$$(x - y)^3 = [(x^3 - y^3) - 3xy(x - y)]$$

So, from equation (i)

Cubing both the sides,

$$\left(x - \frac{y}{2}\right)^3 = (1)^3$$

$$\left(x^3 - \frac{y^3}{8}\right) - 3\left(x \times \frac{y}{2}\right)\left(x - \frac{y}{2}\right) = 1$$

$$[x - \frac{y}{2} = 1] \text{ and } [xy = \frac{3}{2}]$$

$$x^3 - \frac{y^3}{8} - \left(\frac{3}{2} \times \frac{3}{2}\right)(1) = 1$$

$$x^3 - \frac{y^3}{8} - \frac{9}{4} = 1$$

$$x^3 - \frac{y^3}{8} = 1 + \frac{9}{4}$$

$$x^3 - \frac{y^3}{8} = \frac{4+9}{4}$$

$$x^3 - \frac{y^3}{8} = \frac{13}{4}$$

48. If $y^3 + ay^2 + by + 6$ is divisible by $y - 2$ and leaves remainder 3 when divided by $y - 3$, find the values of a and b.

Ans. Let

$$p(y) = y^3 + ay^2 + by + 6$$

$p(y)$ is divisible by $y - 2$

Then $p(2) = 0$

$$23 + a \times 2^2 + b \times 2 + 6 = 0$$

$$8 + 4a + 2b + 6 = 0$$

$$4a + 2b = -14$$

$$2a + b = -7 \quad ...(i)$$

If $p(y)$ is divided by $y - 3$ remainder is 3.

$$p(3) = 3$$

$$3^3 + a \times 3^2 + b \times 3 + 6 = 3$$

$$9a + 3b = -30$$

$$3a + b = -10 \quad ...(ii)$$

Subtracting eq. (i) from eq. (ii)

$$a = -3$$

Put $a = -3$ in eq. (i)

$$2 \times -3 + b = -7$$

$$b = -7 + 6 = -1$$

49. Factorise: $x^3 - 2x^2 - x + 2$.

[Delhi Gov. QB 2022]

Ans. We need to consider the factors of 2, which are ±1 and ±2.

Let us substitute 1 in the polynomial $x^3 - 2x^2 - x + 2$, to get

$(1)^3 - 2(1)^2 - (1) + 2 = 1 - 2 - 1 + 2 = 0$

Thus, according to factor theorem, we can conclude that

$(x - 1)$ is a factor of the polynomial

$x^3 - 2x^2 - x + 2.$

Let us divide the polynomial $x^3 - 2x^2 - x + 2$ by $(x - 1)$, to get

$$\begin{array}{r|l} & x^2 - x - 2 \\ x - 1 & x^3 - 2x^2 - x + 2 \\ & x^3 - x^2 \\ & - \quad + \\ \hline & -x^2 - x \\ & -x^2 + x \\ & + \quad - \\ \hline & -2x + x \\ & -2x + x \\ & + \quad - \\ \hline & 0 \end{array}$$

$$x^3 - 2x^2 - x + 2 = (x - 1)(x^2 - x - 2)$$

$$= (x - 1)(x^2 + x - 2x - 2)$$

$$= (x - 1)[x(x + 1) - 2(x + 1)]$$

$$= (x - 1)(x - 2)(x + 1)$$

Therefore, we can conclude that on factorizing the polynomial $x^3 - 2x^2 - x + 2$, we get

$(x - 1)(x - 2)(x + 1)$